

# Tunable photonic band gaps with coherently driven atoms in optical lattices

David Petrosyan\*

*Institute of Electronic Structure & Laser, FORTH, 71110 Heraklion, Crete, Greece*

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Optical lattice loaded with cold atoms can exhibit a tunable photonic band gap for a weak probe field under the conditions of electromagnetically induced transparency. This system possesses a number of advantageous properties, including reduced relaxation of Raman coherence and the associated probe absorption, and simultaneous enhancement of the index modulation and the resulting reflectivity of the medium. This flexible system has a potential to serve as a testbed of various designs for the linear and nonlinear photonic band gap materials at a very low light level and can be employed for realizing deterministic entanglement between weak quantum fields

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## I. INTRODUCTION

The properties of waves in spatially periodic media are studied in various branches of physics, including acoustics, electromagnetism and quantum mechanics, initially called wave mechanics. Some of the fundamental properties of solid-state systems stem from the interplay of wave-like behavior of amplitudes describing the mobile electrons and the spatially periodic potential created by the crystal lattice, which can result in the forbidden energy bands, or gaps, for the electrons [1]. Analogous effects exist for electromagnetic waves in photonic crystal structures, where the refractive index is a periodic function of spatial coordinates, resulting in the photonic band gaps (PBGs) [2, 3].

Recently, spectacular progress has been achieved in cooling and trapping atoms in the optical lattices (OLs)—spatially periodic dipole potentials induced by off resonant laser fields [4]. The relevant parameters of these systems can be controlled with very high precision and can be tuned to implement with unprecedented accuracy some of the fundamental models of condensed matter physics. A particularly relevant for the present studies achievement has been the demonstration of the transition from the superfluid to the Mott insulator phase with a commensurate number of bosonic atoms per lattice site [5].

As discussed below, OLs loaded with cold atoms interacting with a weak probe field and simultaneously driven by a strong coherent field can serve as a convenient tunable platform for the simulation and studies of light transmission and Bragg reflection in periodic media exhibiting PBGs. Earlier studies of related systems include a weak Bragg reflection of the probe field from the sparsely occupied by atoms 1D OL [6], while more recently, optically induced 1D PBGs in uniform (gaseous) atomic media were realized [7, 8] employing the electromagnetically induced transparency (EIT). EIT is a quantum interference effect characterized by the presence of a

frequency region with greatly reduced absorption accompanied by steep dispersion for a weak probe field propagating in a three-level atomic medium whose adjacent transition is driven by a strong coherent field [9, 10].

Optical lattices loaded with cold atoms under the EIT conditions offer unique advantages over the previously studied schemes. First, the transparency bandwidth and the steep dispersion of the EIT resonance can be easily controlled by the corresponding driving field [9]. Second, for deep enough lattice potential, the Mott insulator regime can be reached [5], in which each site will contain a single tightly localized atom. This will practically eliminate the coherence relaxation on the two-photon Raman transition caused by inter-atomic collisions and atomic time of flight, which result in a residual absorption of weak (quantum) fields and limit interaction times in most EIT experiments. Simultaneously, due to the tight localization of the atoms, the local atomic density is increased by two to three orders of magnitude, as compared to the atomic density in the usual homogeneous (gaseous) media. This, in turn, strongly enhances the modulation amplitude of the effective refractive index, resulting in a strong Bragg reflection of the probe field. The high-contrast periodic index modulation is necessary for achieving 2D and 3D PBGs and strongly trapping the weak probe field. By contrast, in a uniform EIT medium [7], the dispersion modulation with an off-resonant standing wave field can only yield the index contrast up to a few percent, which is sufficient for 1D Bragg reflection over extended propagation lengths amounting to thousands of periods, but not enough for strongly trapping the light in 2D or 3D with several hundred periods in each direction.

The paper is organized as follows. In Sec. II, the polarization of the atomic medium for a weak probe field under the EIT conditions is rigorously derived. This is then used to obtain the 1D coupled-mode equations for the probe field. The solution of these equations and the resulting reflection, transmission and absorption spectra for the probe field in the cold atomic medium are presented in Sec. III. A comparison with the case of a thermal atomic gas is made in Sec. IV. The conclusions are summarized in Sec. V.

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\*E-mail: dap@iesl.forth.gr

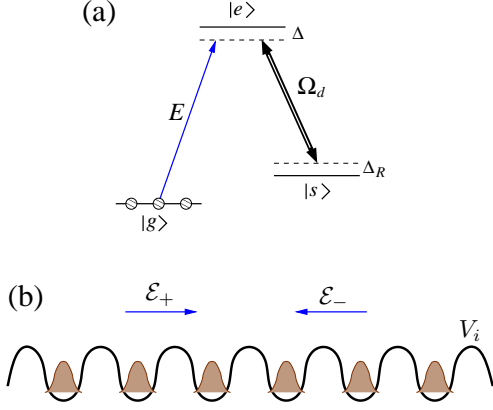


FIG. 1: (a) Level scheme of cold atoms interacting with a weak probe field  $E$  and strong driving field with Rabi frequency  $\Omega_d$ . (b) The atoms are trapped in a tight-binding optical lattice potential  $V_i$ .

## II. MATHEMATICAL FORMALISM

Consider the interaction of a weak probe field with an ensemble of cold bosonic (alkali) atoms with level configuration shown in Fig. 1(a). The probe field  $E$  acts on the transition from the ground  $|g\rangle$  to the excited  $|e\rangle$  state. The transition  $|e\rangle \leftrightarrow |s\rangle$  is driven by a strong (near)resonant field with Rabi frequency  $\Omega_d$ . The atoms in the lower meta-stable states  $|g\rangle$  and  $|s\rangle$  are subject to external spatially periodic potential  $V_{g,s}(\mathbf{r})$  schematically shown in Fig. 1(b). The external potential  $V_e$  for the excited state atoms is assumed negligibly small. All the atoms are initially prepared in the ground state  $|g\rangle$ . At sufficiently low temperatures, only the lowest Bloch band of the lattice potential is populated. In the limit of large lattice depth, this band reduces to the lowest energy levels of individual potential wells. Then, for a lattice filling factor  $\rho \simeq 1$ , and provided that tunneling rate of the atoms between the lattice sites is smaller than on-site interaction energy due to the  $s$ -wave scattering, the Mott-insulator phase with single atom per lattice site will be attained [4, 5].

### A. Polarization of the Cold Atomic Medium

The medium is described by three bosonic field operators  $\hat{\psi}_i(\mathbf{r})$  representing atoms in the corresponding internal state  $|i\rangle$  ( $i = g, e, s$ ) and possessing the standard bosonic commutation relations  $[\hat{\psi}_i(\mathbf{r}), \hat{\psi}_{i'}^\dagger(\mathbf{r}')] = \delta_{ii'} \delta(\mathbf{r} - \mathbf{r}')$  and  $[\hat{\psi}_i(\mathbf{r}), \hat{\psi}_{i'}(\mathbf{r}')] = [\hat{\psi}_i^\dagger(\mathbf{r}), \hat{\psi}_{i'}^\dagger(\mathbf{r}')] = 0$ . In general, the Hamiltonian of the system has the form

$$H = H_A + H_{AA} + H_F + H_{AF}. \quad (1)$$

Here  $H_A$  is the atomic Hamiltonian

$$H_A = \sum_i \int d^3r \hat{\psi}_i^\dagger(\mathbf{r}) \left[ -\frac{\hbar^2}{2M} \nabla^2 + V_i(\mathbf{r}) + \hbar\omega_i \right] \hat{\psi}_i(\mathbf{r}), \quad (2)$$

with  $M$  being the atomic mass and  $\hbar\omega_i$  the internal energy of the atoms in state  $|i\rangle$ . The term  $H_{AA}$  describes the interatomic collisions, which, under the conditions of single atom per lattice site and negligible intersite tunneling assumed above, does not play a role [4, 5]. Next, if the probe field is assumed quantized, its Hamiltonian  $H_F$  must be included in Eq. (1). In the regime of a linear-response of the medium to a weak field studied here, the propagation dynamics of a quantized or a classical field is the same and the probe field can be treated classically. Finally,  $H_{AF}$  describes the interaction of the atoms with the probe  $E(\mathbf{r}, t) \equiv \mathcal{E}(\mathbf{r})e^{-i\omega t}$  and driving  $E_d(\mathbf{r}, t) \equiv \mathcal{E}_d e^{i(\mathbf{k}_d \cdot \mathbf{r} - \omega_d t)}$  fields,

$$H_{AF} = \int d^3r \hat{\psi}_e^\dagger(\mathbf{r}) [-\wp_{eg} E(\mathbf{r}, t)] \hat{\psi}_g(\mathbf{r}) + \int d^3r \hat{\psi}_e^\dagger(\mathbf{r}) [-\wp_{es} E_d(\mathbf{r}, t)] \hat{\psi}_s(\mathbf{r}) + \text{H. c.}, \quad (3)$$

where  $\wp_{ii'}$  is the dipole matrix element of the corresponding atomic transition  $|i\rangle \leftrightarrow |i'\rangle$ , and H. c. stands for the Hermite conjugate. With the Hamiltonian (1), the Heisenberg equations of motion for the atomic operators  $\hat{\psi}_i$  read

$$\frac{\partial}{\partial t} \hat{\psi}_g(\mathbf{r}) = -\frac{i}{\hbar} \left[ -\frac{\hbar^2}{2M} \nabla^2 + V_g(\mathbf{r}) + \hbar\omega_g \right] \hat{\psi}_g(\mathbf{r}) + i \frac{\wp_{ge}}{\hbar} \mathcal{E}^*(\mathbf{r}) e^{i\omega t} \hat{\psi}_e(\mathbf{r}), \quad (4a)$$

$$\frac{\partial}{\partial t} \hat{\psi}_e(\mathbf{r}) = -\frac{i}{\hbar} \left[ -\frac{\hbar^2}{2M} \nabla^2 + \hbar\omega_e \right] \hat{\psi}_e(\mathbf{r}) + i \frac{\wp_{eg}}{\hbar} \mathcal{E}(\mathbf{r}) e^{-i\omega t} \hat{\psi}_g(\mathbf{r}) + i \Omega_d e^{i(\mathbf{k}_d \cdot \mathbf{r} - \omega_d t)} \hat{\psi}_s(\mathbf{r}), \quad (4b)$$

$$\frac{\partial}{\partial t} \hat{\psi}_s(\mathbf{r}) = -\frac{i}{\hbar} \left[ -\frac{\hbar^2}{2M} \nabla^2 + V_s(\mathbf{r}) + \hbar\omega_s \right] \hat{\psi}_s(\mathbf{r}) + i \Omega_d^* e^{-i(\mathbf{k}_d \cdot \mathbf{r} - \omega_d t)} \hat{\psi}_e(\mathbf{r}), \quad (4c)$$

where  $\Omega_d = \wp_{es} \mathcal{E}_d / \hbar$  is the drive Rabi frequency.

The lattice potential is induced by far off-resonant standing-wave fields. It has the form  $V_i(\mathbf{r}) = \hbar \sum_{\xi=x,y,z} S_{i;\xi} \cos^2(k_s \xi)$ , where the modulation (ac Stark shift) amplitude  $S_{i;\xi}$  is proportional to the dynamic polarizability of the corresponding atomic state  $|i\rangle$  ( $i = g, s$ ) and the intensity of the standing-wave fields with the wave vector  $k_s$ . As stated above, in a deep OL potential, the atomic wavefunction is tightly localized at each lattice site  $j$ . The atomic field operators  $\hat{\psi}_i(\mathbf{r})$  can then be expanded in terms of the real, normalized (Wannier) functions  $w_i^{(j)}(\mathbf{r}) = w_i(\mathbf{r} - \mathbf{r}_j)$ , which in the vicinity of  $\mathbf{r} \sim \mathbf{r}_j$  satisfy

$$\left[ -\frac{\hbar^2}{2M} \nabla^2 + V_i(\mathbf{r}) \right] w_i^{(j)}(\mathbf{r}) = \hbar\nu_i w_i^{(j)}(\mathbf{r}), \quad (5)$$

where  $\hbar\nu_i$  is the energy of the lowest vibrational state of the nearly harmonic potential well, while  $w_i^{(j)}(\mathbf{r})$  can well be approximated by 3D Gaussian functions

$$w_i^{(j)}(\mathbf{r}) = \left(\frac{1}{\pi\delta r^2}\right)^{3/4} \exp\left[-\frac{(\mathbf{r}-\mathbf{r}_j)^2}{2\delta r^2}\right], \quad (6)$$

centered around  $\mathbf{r}_j$  with the width  $\delta r$ . For tight localization  $\delta r \ll \pi/k_s$ , the Wannier functions pertaining to different lattice sites have negligible overlap,  $\int d^3r w_i^{(j)}(\mathbf{r}) w_i^{(j')}(\mathbf{r}) = \delta_{jj'}$ . The field operators for the lower atomic states can then be decomposed as  $\hat{\psi}_g(\mathbf{r}, t) = \sum_j w_g^{(j)}(\mathbf{r}) \hat{g}_j(t) e^{-i(\omega_g + \nu_g)t}$  and  $\hat{\psi}_s(\mathbf{r}, t) = \sum_j w_s^{(j)}(\mathbf{r}) \hat{s}_j(t) e^{-i(\omega_s + \nu_s)t} e^{-i(\omega - \omega_d)t}$ , where  $\hat{g}_j(t)$  and  $\hat{s}_j(t)$  are the slowly-varying in time annihilation operators for the bosonic atoms at site  $j$ . Since the atoms in the excited state  $|e\rangle$  are assumed free,  $V_e \simeq 0$ , the corresponding atomic field operator can be expanded as  $\hat{\psi}_e(\mathbf{r}, t) = \sum_{\mathbf{k}_l} u_{\mathbf{k}_l}(\mathbf{r}) \hat{e}_{\mathbf{k}_l}(t) e^{-i(\omega_e + \nu_e)t} e^{-i\omega t}$ , where  $u_{\mathbf{k}_l}(\mathbf{r}) = (L)^{-3/2} e^{i\mathbf{k}_l \cdot \mathbf{r}}$  are the plane waves within the quantization (medium) volume  $L^3$ , which satisfy

$$-\frac{\hbar^2}{2M} \nabla^2 u_{\mathbf{k}_l}(\mathbf{r}) = \hbar\nu_{k_l} u_{\mathbf{k}_l}(\mathbf{r}), \quad (7)$$

$$\int d^3r u_{\mathbf{k}_l}^*(\mathbf{r}) u_{\mathbf{k}_l'}(\mathbf{r}) = \delta_{\mathbf{k}_l \mathbf{k}_l'}, \quad \hbar\nu_{k_l} = \frac{\hbar^2 k_l^2}{2M}, \quad (8)$$

and  $\hat{e}_{\mathbf{k}_l}(t)$  are the slowly-varying in time annihilation operators for the corresponding mode.

Substituting the above decompositions of the atomic field operators into Eqs. (4) yields

$$\begin{aligned} \sum_j w_g^{(j)}(\mathbf{r}) \dot{\hat{g}}_j &= -\gamma_g \sum_j w_g^{(j)}(\mathbf{r}) \hat{g}_j \\ &+ i \frac{\wp_{ge}}{\hbar} \mathcal{E}^*(\mathbf{r}) \sum_{\mathbf{k}_l} u_{\mathbf{k}_l}(\mathbf{r}) \hat{e}_{\mathbf{k}_l} + \hat{\mathcal{F}}_g, \end{aligned} \quad (9a)$$

$$\begin{aligned} \sum_{\mathbf{k}_l} u_{\mathbf{k}_l}(\mathbf{r}) \dot{\hat{e}}_{\mathbf{k}_l} &= \sum_{\mathbf{k}_l} (i\Delta - i\nu_{k_l} - \gamma_e) u_{\mathbf{k}_l}(\mathbf{r}) \hat{e}_{\mathbf{k}_l} \\ &+ i \frac{\wp_{eg}}{\hbar} \mathcal{E}(\mathbf{r}) \sum_j w_g^{(j)}(\mathbf{r}) \hat{g}_j \\ &+ i\Omega_d e^{i\mathbf{k}_d \cdot \mathbf{r}} \sum_j w_s^{(j)}(\mathbf{r}) \hat{s}_j + \hat{\mathcal{F}}_e, \end{aligned} \quad (9b)$$

$$\begin{aligned} \sum_j w_s^{(j)}(\mathbf{r}) \dot{\hat{s}}_j &= (i\Delta_R - \gamma_s) \sum_j w_s^{(j)}(\mathbf{r}) \hat{s}_j \\ &+ i\Omega_d^* e^{-i\mathbf{k}_d \cdot \mathbf{r}} \sum_{\mathbf{k}_l} u_{\mathbf{k}_l}(\mathbf{r}) \hat{e}_{\mathbf{k}_l} + \hat{\mathcal{F}}_s, \end{aligned} \quad (9c)$$

where  $\Delta \equiv \omega + \omega_g + \nu_g - \omega_e$  and  $\Delta_R \equiv \omega + \omega_g + \nu_g - \omega_s - \nu_s - \omega_d$  are, respectively, the one-photon and two-photon (Raman) detunings of the probe field [cf. Fig. 1(a)], while  $\gamma_i$  are the phenomenological half-decay rates of the corresponding atomic states  $|i\rangle$  ( $i = g, e, s$ ). Note that  $\gamma_e \sim 10^7 \text{ s}^{-1}$  is large, determined by the short radiative lifetime of the excited atomic state, while for isolated cold atoms trapped in a conservative OL potential,  $\gamma_{g,s} \lesssim 1 \text{ s}^{-1}$  are negligibly small, due to the absence

of interatomic collisions and atomic free-flight or diffusion away from the interaction region. Finally,  $\hat{\mathcal{F}}_i$  are the  $\delta$ -correlated noise operators associated with the relaxation. These noise operators, required to preserve the formal consistency of the equations for the atomic operators, will not play a role in the following semiclassical treatment of the field propagation and will be dropped from now on.

Under the weak probe approximation, the initial atomic population in the ground state  $|g\rangle$  remains practically undepleted and Eqs. (9) can be solved to the lowest (first) order in  $\mathcal{E}(\mathbf{r})$ . In the stationary regime, Eq. (9c) yields

$$\sum_j w_s^{(j)}(\mathbf{r}) \hat{s}_j = -i \frac{\Omega_d^* e^{-i\mathbf{k}_d \cdot \mathbf{r}} \sum_{\mathbf{k}_l} u_{\mathbf{k}_l}(\mathbf{r}) \hat{e}_{\mathbf{k}_l}}{i\Delta_R - \gamma_s}.$$

Substituting this into Eq. (9b), multiplying all terms by  $\int d^3r u_{\mathbf{k}_l}^*(\mathbf{r})$  and using Eq. (8) leads to

$$\hat{e}_{\mathbf{k}_l} = -i \frac{\wp_{eg}}{\hbar} \frac{\sum_j \int d^3r u_{\mathbf{k}_l}^*(\mathbf{r}) \mathcal{E}(\mathbf{r}) w_g^{(j)}(\mathbf{r}) \hat{g}_j}{i\Delta - \gamma_e + \frac{|\Omega_d|^2}{i\Delta_R - \gamma_s} - i\nu_{k_l}}. \quad (10)$$

The integral in the above equation can be evaluated assuming that the localization width  $\delta r$  of  $w_g^{(j)}(\mathbf{r})$  is small compared to the wavelength  $\lambda = 2\pi c/\omega$  of the probe field. Therefore  $\mathcal{E}(\mathbf{r})$  changes little in the vicinity of  $\mathbf{r} \simeq \mathbf{r}_j$  and can be taken constant in this region. The remaining integral is then given by

$$\begin{aligned} \mathcal{I}_{\mathbf{k}_l}^{(j)} &= \int d^3r u_{\mathbf{k}_l}^*(\mathbf{r}) w_g^{(j)}(\mathbf{r}) \\ &= \left(\frac{1}{\pi\delta r^2}\right)^{3/4} (\sqrt{2\pi}\delta r)^3 u_{\mathbf{k}_l}^*(\mathbf{r}_j) e^{-\delta r^2 \mathbf{k}_l^2/2}. \end{aligned} \quad (11)$$

The polarization  $P(\mathbf{r}, t) = \mathcal{P}(\mathbf{r}) e^{-i\omega t}$  of the atomic medium, induced by the applied probe field  $E(\mathbf{r}, t)$ , is given by the expectation value of the atomic dipole moment at position  $\mathbf{r}$  and time  $t$ ,  $P(\mathbf{r}, t) = \langle \hat{\psi}_g^\dagger(\mathbf{r}, t) \wp_{ge} \hat{\psi}_e(\mathbf{r}, t) \rangle$ . With the above decompositions of the atomic field operators  $\hat{\psi}_{g,e}$  and using Eq. (10), the medium polarization  $\mathcal{P}(\mathbf{r})$  reads

$$\begin{aligned} \mathcal{P}(\mathbf{r}) &= \wp_{ge} \left\langle \sum_j w_g^{(j)}(\mathbf{r}) \hat{g}_j^\dagger \sum_{\mathbf{k}_l} u_{\mathbf{k}_l}(\mathbf{r}) \hat{e}_{\mathbf{k}_l} \right\rangle \\ &= -i \frac{|\wp_{eg}|^2}{\hbar} \sum_{j,j'} w_g^{(j)}(\mathbf{r}) \mathcal{E}(\mathbf{r}_{j'}) \langle \hat{g}_j^\dagger \hat{g}_{j'} \rangle \\ &\quad \times \sum_{\mathbf{k}_l} \frac{u_{\mathbf{k}_l}(\mathbf{r}) \mathcal{I}_{\mathbf{k}_l}^{(j')}}{i\Delta - \gamma_e + \frac{|\Omega_d|^2}{i\Delta_R - \gamma_s} - i\nu_{k_l}}. \end{aligned} \quad (12)$$

Here the sum over the plane-wave modes  $\mathbf{k}_l$  should be replaced by an integral according to  $\sum_{\mathbf{k}_l} \rightarrow (L/2\pi)^3 \int d^3k_l$

[10], which, upon substituting  $\mathcal{I}_{\mathbf{k}_l}^{(j')}$  from Eq. (11), reads

$$\left(\frac{L}{2\pi}\right)^3 \left(\frac{1}{\pi\delta r^2}\right)^{3/4} (\sqrt{2\pi}\delta r)^3 \times \int d^3k_l \frac{u_{\mathbf{k}_l}(\mathbf{r}) u_{\mathbf{k}_l}^*(\mathbf{r}_{j'}) e^{-\delta r^2 \mathbf{k}_l^2/2}}{i\Delta - \gamma_e + \frac{|\Omega_d|^2}{i\Delta_R - \gamma_s} - i\nu_{k_l}}. \quad (13)$$

It is easy to see that if the atomic localization width is not too small,  $\delta r^2 \gg \delta r_{\min}^2 = \hbar/M\gamma_e$  [11],  $\nu_{k_l} = \frac{\hbar k_l^2}{2M}$  in the denominator of the above equation contributes little to the integral and can therefore be dropped. The integral is then easily evaluated, and Eq. (13) reduces to

$$\frac{w_g^{(j')}(\mathbf{r})}{i\Delta - \gamma_e + \frac{|\Omega_d|^2}{i\Delta_R - \gamma_s}}.$$

Since the Wannier functions  $w_g^{(j)}(\mathbf{r})$  and  $w_g^{(j')}(\mathbf{r})$  pertaining to different lattice sites  $j \neq j'$  have negligible overlap, the final expression for the medium polarization takes the form

$$\mathcal{P}(\mathbf{r}) = i \frac{|\wp_{eg}|^2}{\hbar} \frac{\varrho(\mathbf{r}) \mathcal{E}(\mathbf{r})}{\gamma_e - i\Delta + \frac{|\Omega_d|^2}{\gamma_s - i\Delta_R}}, \quad (14)$$

where  $\varrho(\mathbf{r}) = \sum_j |w_g^{(j)}(\mathbf{r})|^2 \langle \hat{g}_j^\dagger \hat{g}_j \rangle$  is the spatially-periodic density of the medium. According to the assumption above, the occupation number of each lattice site is  $\langle \hat{g}_j^\dagger \hat{g}_j \rangle = 1$ . Clearly, the spectral response of the medium coincides with that of the conventional EIT with homogeneous atomic ensemble [9, 10], but is spatially modulated by the periodic atomic density, which leads to profound consequences for the probe transmission and reflection, as discussed below.

## B. Coupled-Mode Equations for the Probe Field

The propagation of electromagnetic field in the medium is governed by the Maxwell wave equation

$$\left[\nabla^2 - \mu_0\epsilon_0 \frac{\partial^2}{\partial t^2}\right] E(\mathbf{r}, t) = \mu_0 \frac{\partial^2}{\partial t^2} \mathcal{P}(\mathbf{r}, t). \quad (15)$$

In free space,  $P = 0$ , its general solution for a monochromatic field of frequency  $\omega$  has the form  $E(\mathbf{r}, t) = \sum_{\mathbf{k}} \mathcal{E}_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ , where  $\mathcal{E}_{\mathbf{k}}$  is the amplitude of the field mode with wave vector  $\mathbf{k}$  satisfying  $|\mathbf{k}| = \omega/c$ , with  $c = (\mu_0\epsilon_0)^{-1/2}$  being the speed of light in vacuum.

Consider a stationary propagation of the probe field in the periodic atomic medium whose spatial and spectral properties are characterized by Eq. (14). In particular, assume that a circularly ( $\sigma^-$ ) polarized probe field propagates along the  $\hat{\mathbf{z}}$  axis (taken to be the quantization axis), while a linearly ( $\pi$ ) polarized driving field, propagating along the  $\hat{\mathbf{x}}$  axis ( $\mathbf{k}_d \perp \hat{\mathbf{z}}$ ), uniformly irradiates the whole atomic sample. Following the approach of [3], the probe

field can be expanded in terms of the free-space normal modes as  $E(z, t) = \sum_{\mathbf{k}} \mathcal{E}_{\mathbf{k}}(z) e^{i(kz - \omega t)}$ , where  $\mathcal{E}_{\mathbf{k}}(z)$  are now slowly varying in space amplitudes of the corresponding field modes  $k = \pm\omega/c$ . The medium polarization can be expressed as  $P(z, t) = \epsilon_0 \chi(\omega; z) E(z, t)$ , where  $\chi(\omega; z)$  is the linear susceptibility, whose  $z$ -dependence is due to the atomic density  $\varrho(z)$ . This 1D density is obtained by averaging  $\varrho(\mathbf{r})$  over the transverse to  $\hat{\mathbf{z}}$  directions of the cubic lattice with primitive vector  $\Lambda = \pi/k_s$ ,

$$\varrho(z) \equiv \frac{1}{\Lambda^2} \int_{-\Lambda/2}^{\Lambda/2} dx \int_{-\Lambda/2}^{\Lambda/2} dy \varrho(\mathbf{r}). \quad (16)$$

Since  $\varrho(z) = \varrho(z + \Lambda)$  is a periodic function, it can be expanded in a Fourier series

$$\varrho(z) = \sum_l \varrho_l e^{ilgz}, \quad g = \frac{2\pi}{\Lambda} = 2k_s, \quad (17)$$

with the expansion coefficients

$$\begin{aligned} \varrho_l &= \frac{1}{\Lambda} \int_{-\Lambda/2}^{\Lambda/2} dz \varrho(z) e^{-ilgz} \\ &= \frac{\langle \hat{g}_j^\dagger \hat{g}_j \rangle}{\Lambda^3} \int_{-\Lambda/2}^{\Lambda/2} dz |w_g(z)|^2 e^{-ilgz}, \end{aligned} \quad (18)$$

where, according to Eq. (6),

$$|w_g(z)|^2 = \frac{1}{\sqrt{\pi}\delta r} \exp\left[-\frac{z^2}{\delta r^2}\right] \quad (19)$$

is a normalized Gaussian function. Thus, the zeroth Fourier component  $\varrho_0 = \langle \hat{g}_j^\dagger \hat{g}_j \rangle / \Lambda^3$  ( $\delta r \ll \Lambda$ ) is the spatially averaged (bulk) atomic density. Higher order Fourier components can be expressed as  $\varrho_l = \varrho_0 \kappa_l$ , where

$$\kappa_l \equiv \int_{-\Lambda/2}^{\Lambda/2} dz |w_g(z)|^2 e^{-ilgz}. \quad (20)$$

Using Eq. (14), the medium polarization then reads

$$P(z, t) = \epsilon_0 \chi(\omega) \sum_l \sum_{k'} \kappa_l \mathcal{E}_{k'}(z) e^{i(lg+k')z} e^{-i\omega t}, \quad (21)$$

where

$$\chi(\omega) \equiv \chi_{\text{EIT}}(\omega) = 2 \frac{c}{\omega} \sigma_0 \varrho_0 \frac{i\gamma_e}{\gamma_e - i\Delta + \frac{|\Omega_d|^2}{\gamma_s - i\Delta_R}}, \quad (22)$$

is the usual EIT susceptibility of an atomic medium with uniform density  $\varrho_0$  and resonant absorption cross-section  $\sigma_0 = |\wp_{eg}|^2 \omega_{eg} / (2\epsilon_0 c \hbar \gamma_e)$  [10]. By definition, the resonant amplitude absorption coefficient in the two-level atomic medium ( $\Omega_d = 0$ ) is  $a_0 = \sigma_0 \varrho_0$ .

Using Eq. (21), Maxwell's equation (15) yields

$$\begin{aligned} 2i \sum_k k \left[ \frac{d}{dz} \mathcal{E}_k(z) \right] e^{ikz} \\ = -\frac{\omega^2}{c^2} \chi(\omega) \sum_l \sum_{k'} \kappa_l \mathcal{E}_{k'}(z) e^{i(lg+k')z}, \end{aligned} \quad (23)$$

which, upon expanding the sums over  $k, k' = \pm\omega/c$ , leads to the following coupled equations for the amplitudes  $\mathcal{E}_\pm \equiv \mathcal{E}_{\pm\omega/c}$  of the forward (+) and backward (-) propagating modes,

$$\frac{d}{dz}\mathcal{E}_+ = i\alpha(\omega) \sum_l \kappa_l [\mathcal{E}_+ e^{ilgz} + \mathcal{E}_- e^{i(lg-2\omega/c)z}], \quad (24a)$$

$$\frac{d}{dz}\mathcal{E}_- = -i\alpha(\omega) \sum_l \kappa_l [\mathcal{E}_+ e^{i(lg+2\omega/c)z} + \mathcal{E}_- e^{ilgz}], \quad (24b)$$

where

$$\alpha(\omega) \equiv \frac{\omega}{c} \frac{\chi(\omega)}{2} = \frac{ia_0\gamma_e}{\gamma_e - i\Delta + \frac{|\Omega_d|^2}{\gamma_s - i\Delta_R}}. \quad (25)$$

Note that  $\kappa_l$  decrease as  $|l|$  increase. Thus, apart from  $\kappa_0 = 1$ ,  $\kappa_{\pm 1}$  are the largest coefficients given by ( $\delta r \ll \Lambda$ )

$$\kappa_{\pm 1} \simeq \frac{1}{\sqrt{\pi}\delta r} \int_{-\infty}^{\infty} dz e^{-z^2/\delta r^2} \cos(2\pi z/\Lambda). \quad (26)$$

Assuming, therefore, that  $\omega/c \sim k_s$  and keeping only the terms satisfying the longitudinal phase matching condition (i.e., slowly oscillating in space), Eqs. (24) reduce to

$$\frac{d}{dz}\mathcal{E}_+ = i\alpha(\omega)\mathcal{E}_+ + i\alpha(\omega)\kappa_1\mathcal{E}_- e^{-2i(\omega/c-k_s)z}, \quad (27a)$$

$$\frac{d}{dz}\mathcal{E}_- = -i\alpha(\omega)\mathcal{E}_- - i\alpha(\omega)\kappa_{-1}\mathcal{E}_+ e^{2i(\omega/c-k_s)z}. \quad (27b)$$

In these equations, the first term describes the usual EIT absorption and dispersion of the probe field propagating in the atomic medium, while the second term is responsible for the coupling of the forward and backward propagating modes mediated by the medium periodicity.

### III. REFLECTION, TRANSMISSION AND ABSORPTION OF THE PROBE

The coupled mode equations (27) fully determine the optical properties of the system under consideration. The solution of Eqs. (27) for the boundary value problem defined through  $\mathcal{E}_+(0) = \mathcal{E}_{\text{in}}$  and  $\mathcal{E}_-(L) = 0$  is given by

$$\mathcal{E}_+(z) = \mathcal{E}_{\text{in}} e^{-i(\omega/c-k_s)z} \times \frac{s \cosh[s(L-z)] - i\delta\beta \sinh[s(L-z)]}{s \cosh[sL] - i\delta\beta \sinh[sL]}, \quad (28a)$$

$$\mathcal{E}_-(z) = \mathcal{E}_{\text{in}} e^{i(\omega/c-k_s)z} \times \frac{i\alpha(\omega)\kappa_{-1} \sinh[s(L-z)]}{s \cosh[sL] - i\delta\beta \sinh[sL]}, \quad (28b)$$

where the coefficients

$$\delta\beta = \alpha(\omega) + \frac{\omega}{c} - k_s, \\ s = \sqrt{\alpha^2(\omega)\kappa_1\kappa_{-1} - \delta\beta^2},$$

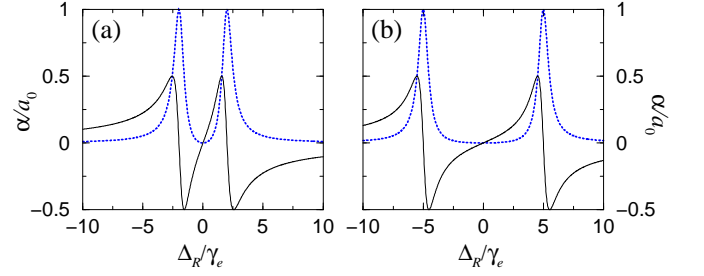


FIG. 2: Absorption (blue dotted lines) and dispersion (black solid lines) spectra for the probe field in a uniform EIT atomic medium. The Rabi frequency of the resonant driving field is (a)  $\Omega_d = 2\gamma_e$ , and (b)  $\Omega_d = 5\gamma_e$ .

quantify, respectively, the phase mismatch and the coupling between the forward and backward propagating modes.

According to the Bloch theorem [2, 3], the general solution for the electromagnetic field propagation in the periodic medium can be cast as  $\mathcal{E}(z) = e^{iKz}\mathcal{E}_K(z)$ , where  $K$  is a propagation constant known as the Bloch wave vector, while  $\mathcal{E}_K(z) = \mathcal{E}_K(z + \Lambda)$  is a spatially periodic function. In a purely dispersive (non-absorbing) medium, the real and imaginary parts of  $K$  describe, respectively, the spatial phase and the attenuation (due to Bragg reflection) of the field upon propagation. It follows from Eqs. (28) that the corresponding propagation constant for the monochromatic probe field of frequency  $\omega$  is given by

$$K = k_s + is \\ = k_s + i\sqrt{[\alpha(\omega)\kappa_1]^2 - \left[\alpha(\omega) + \frac{\omega}{c} - k_s\right]^2}. \quad (29)$$

In the present situation, the periodic atomic medium exhibits both strong dispersion and absorption characterized by the EIT polarizability  $\alpha(\omega)$  (see Fig. 2). Therefore, in the dispersion relation of Eq. (29), both Bragg reflection and medium absorption would contribute to the imaginary part of  $K$ . A more quantitative and experimentally accessible characterization of the optical properties of the system is provided by the reflection  $R$ , transmission  $T$  and absorption  $A$  coefficients for the probe field, which are defined through

$$R(\omega) \equiv \left| \frac{\mathcal{E}_-(0)}{\mathcal{E}_+(0)} \right|^2, \quad (30a)$$

$$T(\omega) \equiv \left| \frac{\mathcal{E}_+(L)}{\mathcal{E}_+(0)} \right|^2, \quad (30b)$$

$$A(\omega) = 1 - [R(\omega) + T(\omega)]. \quad (30c)$$

The absorption and dispersion spectra for the probe field in a uniform atomic medium subject to a resonant drive  $\omega_d = \omega_{es} - \nu_s$  is shown in Fig. 2. The strong driving field with Rabi frequency  $\Omega_d \gtrsim \gamma_e$  splits the familiar Lorentzian absorption line into the Autler-Townes doublet with the two peaks separated by  $2\Omega_d$ , while at the

line center the medium becomes transparent to the probe field. This EIT is accompanied by a steep normal dispersion, resulting in the greatly reduced group velocity [9, 10]

$$v_g = \frac{c}{1 + c \frac{\partial}{\partial \omega} \text{Re } \alpha(\omega)} \simeq \frac{|\Omega_d|^2}{a_0 \gamma_e} \ll c. \quad (31)$$

The principal aim here is to explore the possibility of a PBG for the probe field in the frequency region of the reduced absorption. Assuming resonant driving field, so that  $\Delta_R = \Delta$ , in the vicinity of EIT resonance  $|\Delta| < \Omega_d$ , the polarizability of Eq. (25) reduces to  $\alpha(\omega) \simeq \Delta/v_g$  and the dispersion relation (29) can be approximated by

$$K - k_s \simeq i \sqrt{\left[ \frac{\Delta}{v_g \kappa_1} \right]^2 - \left[ \frac{\Delta}{v_g} - \frac{\delta_s}{c} \right]^2}, \quad (32)$$

where  $\delta_s \equiv k_s c - \omega_{eg}$ . The Bloch wave vector  $K$  has an imaginary part when the term under the square-root of this equation is positive. The corresponding range of frequencies of the probe field is

$$\frac{\delta_s v_g}{(1 \pm \kappa_1)c} < \Delta < \frac{\delta_s v_g}{(1 \mp \kappa_1)c}, \quad \text{for } \delta_s \gtrless 0. \quad (33)$$

Thus, a PBG of width  $\delta\omega_{\text{gap}} = |\delta_s| \frac{v_g}{c} \frac{2\kappa_1}{1-\kappa_1^2}$ , centered at  $\Delta_{\text{gap}} = \delta_s \frac{v_g}{c} \frac{1}{1-\kappa_1^2}$ , is formed on the positive (for  $\delta_s > 0$ ) or negative (for  $\delta_s < 0$ ) side of the EIT resonance. The peak value of the imaginary part of  $K$ , which determines the maximal reflectivity at  $\Delta_{\text{gap}}$ , is given by

$$\max(\text{Im } K) = \frac{|\delta_s|}{c} \frac{\kappa_1}{\sqrt{1-\kappa_1^2}}. \quad (34)$$

These conclusions are justified upon requiring that the PBG lies within the EIT window  $\delta\omega_{\text{tw}} = |\Omega_d|^2/(\gamma_e \sqrt{2a_0 L})$  [10, 12] where the absorption is small. This leads to the following condition on the lattice wavevector  $k_s$ ,

$$|k_s - \omega_{eg}/c| \equiv \frac{|\delta_s|}{c} \lesssim (1 - \kappa_1^2) \sqrt{\frac{a_0}{2L}}, \quad (35)$$

with which  $\max(\text{Im } K) \lesssim \kappa_1 \sqrt{a_0(1 - \kappa_1^2)/2L}$ . With the system parameters listed in [11], and for an atomic medium of length  $L \simeq 200 \mu\text{m}$  (500 lattice periods  $\Lambda$ ), or optical depth  $2a_0 L = 100$ , the above condition is satisfied for  $|k_s - \omega_{eg}/c| \simeq 4.5 \times 10^3 \text{ rad/m}$  or  $|\delta_s|/2\pi \simeq 2.16 \times 10^{11} \text{ s}^{-1}$ . Physically, such a large difference between the lattice wave vector  $k_s$  and that of the probe field  $k \simeq \omega_{eg}/c$  stems from the need to satisfy the phase matching condition  $\delta\beta \simeq 0$  [while  $\alpha(\omega)\kappa_1 \neq 0$ ] in strongly dispersive EIT medium.

The complete dispersion relation of Eq. (29), with  $\kappa_1 \simeq 0.9$  obtained for  $\delta r \simeq \Lambda/10$  [11], is plotted in Fig. 3 for the two values of  $\Omega_d$  used in Fig. 2. As noted above, the imaginary part of  $K$  describes exponential

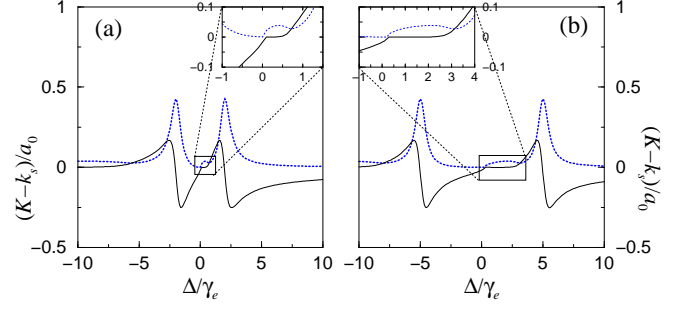


FIG. 3: Imaginary (blue dotted lines) and real (black solid lined) parts of  $K$  as a function of probe detuning  $\Delta$  (resonant drive,  $\Delta = \Delta_R$ ), for  $\kappa_1 = 0.9$  ( $\delta r \simeq \Lambda/10$ ),  $\delta_s = 7.2 \times 10^4 \gamma_e$ , and (a)  $\Omega_d = 2\gamma_e$  and (b)  $\Omega_d = 5\gamma_e$ . Insets magnify the important frequency regions within the EIT window.

attenuation of the forward propagating probe field due to the back reflection and medium absorption. Within the transparency window,  $|\Delta| \lesssim \delta\omega_{\text{tw}}$ , the absorption is much smaller than the spatially-periodic dispersion, and the bulges of  $\text{Im } K$  in the insets of Fig. 3 signify the appearance of the PBG. The band edges given by Eq. (33) can be tuned by changing the drive field intensity, since  $v_g \propto |\Omega_d|^2$ .

Figure 4 shows the reflection, transmission and absorption spectra for the probe field for the two values of  $\Omega_d$  used in Figs. 2 and 3. The peak reflectivity for the PBG within the EIT window is about 90%, limited mainly by absorption. Choosing smaller values of  $\delta_s$ , thereby moving the PBG closer to the EIT line center, and taking si-

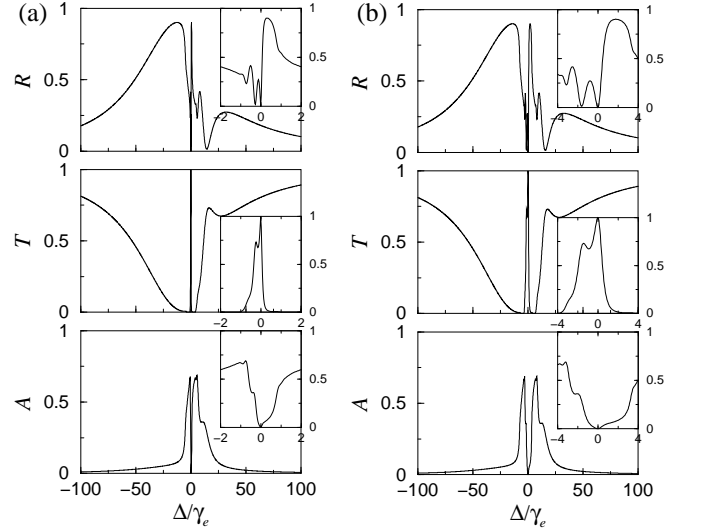


FIG. 4: Reflection  $R$ , transmission  $T$  and absorption  $A$  spectra for the probe field in a cold EIT atomic medium loaded into an OL. The medium length  $L = 500\Lambda \simeq 200 \mu\text{m}$ , or optical depth  $2a_0 L = 100$ , and all the other parameters are same as in Fig. 3(a) and (b), respectively. Insets magnify the important frequency regions within the EIT window.

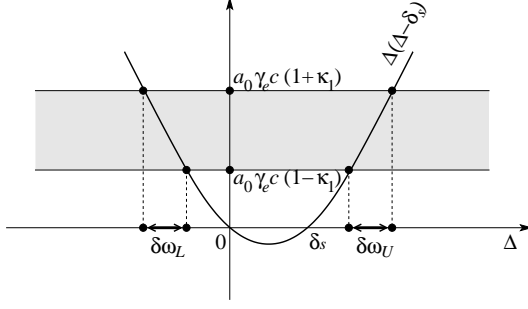


FIG. 5: Diagram illustrating the appearance of the lower  $\delta\omega_L$  and upper  $\delta\omega_U$  PBGs far away from the EIT resonance, where the atoms behave as off-resonant two-level systems with negligible absorption.

multaneously longer medium, to compensate for smaller values of  $\max(\text{Im } K)$  as per Eq. (34), will yield even larger values of the maximum reflection (up to 98%), at the expense of narrower PBG.

In the spectral region far away from the atomic line center and EIT features, there is another broad PBG seen in the reflection spectrum of Fig. 4. Its existence was predicted and studied in [6] for the limiting case of  $\kappa_1 \rightarrow 1$ , corresponding to the atomic localization width  $\delta r \rightarrow 0$ . Briefly, in the frequency regions  $|\Delta| \gg \gamma_e, \Omega_d$ , the polarizability of Eq. (25) can be approximated as  $\alpha(\omega) \simeq -a_0\gamma_e/\Delta$ , which describes the response of off-resonant two-level atoms, and the dispersion relation reads

$$K - k_s \simeq i\sqrt{\left[\frac{a_0\gamma_e}{\Delta}\kappa_1\right]^2 - \left[\frac{a_0\gamma_e}{\Delta} + \frac{\delta_s - \Delta}{c}\right]^2}. \quad (36)$$

The Bloch wave vector acquires imaginary part when  $a_0\gamma_e c(1 - \kappa_1) < \Delta(\Delta - \delta_s) < a_0\gamma_e c(1 + \kappa_1)$ , which is graphically illustrated in Fig. 5. Two PBGs appear, one on the red  $\Delta < 0$  and the other on the blue  $\Delta > 0$  sides of the atomic resonance. The corresponding ranges of frequencies for the lower (L) and upper (U) gaps are

$$\frac{1}{2}\delta_s - D_+ < \Delta_L < \frac{1}{2}\delta_s - D_-, \quad (37a)$$

$$\frac{1}{2}\delta_s + D_- < \Delta_U < \frac{1}{2}\delta_s + D_+, \quad (37b)$$

where  $D_{\pm} \equiv \sqrt{(\delta_s/2)^2 + a_0\gamma_e c(1 \pm \kappa_1)}$ , and the widths of the gaps are  $\delta\omega_{L,U} = D_+ - D_-$ . When  $(\delta_s/2)^2 \gg a_0\gamma_e c$ , which for  $\delta_s$  chosen as in Eq. (35) is certainly satisfied,  $D_{\pm}$  can be expanded as  $D_{\pm} \simeq |\delta_s|/2 + a_0\gamma_e c(1 \pm \kappa_1)/|\delta_s|$ . For  $\delta_s > 0$ , the lower gap  $-a_0\gamma_e c(1 + \kappa_1)/|\delta_s| < \Delta_L < -a_0\gamma_e c(1 - \kappa_1)/|\delta_s|$  is closer to the atomic resonance, and the corresponding reflection is seen in Fig. 4; the upper gap is beyond the range of frequencies spanned in that figure. Note finally that the reflection, transmission and absorption spectra for  $\delta_s > 0$  and  $\delta_s < 0$  are related by mirror symmetry about  $\Delta = 0$ .

#### IV. THERMAL ATOMIC MEDIUM

For the sake of comparison, it is instructive to consider the case of a thermal atomic gas under the otherwise similar conditions. When the thermal energy  $k_B T$  is much larger than the depth of the OL potential  $\hbar S_i$ , the atoms move freely and the atomic density  $\rho$  is uniform [13]. However, the off-resonant standing wave field with wave-vector  $k_s$  results in a spatially-periodic ac Stark-shift  $\hbar S_i \cos^2(k_s z)$  of the lower atomic levels  $|i\rangle$  ( $i = g, s$ ). The resulting susceptibility

$$\chi(\omega; z) = 2 \frac{c}{\omega} \frac{ia_0\gamma_e}{\gamma_e - i\Delta(z) + \frac{|\Omega_d|^2}{\gamma_s - i\Delta_R(z)}}, \quad (38)$$

is a periodic function of  $z$ , since the corresponding one- and two-photon detunings  $\Delta(z) = \Delta' + \frac{1}{2}S_g \cos(2k_s z)$  and  $\Delta_R(z) = \Delta'_R + \frac{1}{2}S_{gs} \cos(2k_s z)$  are ac Stark modulated. Here  $\Delta' = \omega - \omega_{eg} + \frac{1}{2}S_g$  and  $\Delta'_R = \omega - \omega_d + \omega_{gs} + \frac{1}{2}S_{gs}$  are the mean detunings, while  $S_{gs} = S_g - S_s$  is the difference of the ac Stark modulation amplitudes for levels  $|g\rangle$  and  $|s\rangle$ . The decay rate  $\gamma_s$  describes the Raman coherence relaxation, which is now affected by the atomic motion and collisions; its typical value is in the range of  $\gamma_s \sim 10^3 - 10^4 \text{ s}^{-1}$  [9], which is still much smaller than  $\gamma_e$  but not negligible. The effect of the spatially-periodic modulation of the Raman detuning  $\Delta_R(z)$  is to periodically shift the EIT spectrum for the probe field (see Fig. 2), which under appropriate conditions discussed below can result in a PBG, as was shown in [7].

Substituting the susceptibility of Eq. (38) into the Maxwell equation (15) with  $P(z, t) = \epsilon_0 \chi(\omega; z) E(z, t)$  and  $E(z, t) = \sum_k \mathcal{E}_k(z) e^{i(kz - \omega t)}$ , under the EIT conditions  $|\Omega_d|^2 \gg \gamma_e \gamma_s, (\frac{1}{4}S_{g,s})^2$  and the longitudinal phase matching  $k_s \simeq \omega/c$ , the following coupled mode equations for the forward and backward propagating modes  $k = \pm\omega/c$  of the probe field are obtained,

$$\frac{d}{dz} \mathcal{E}_+ = i\alpha'(\omega) \mathcal{E}_+ + i\eta(\omega) \mathcal{E}_- e^{-2i(\omega/c - k_s)z}, \quad (39a)$$

$$\frac{d}{dz} \mathcal{E}_- = -i\alpha'(\omega) \mathcal{E}_- - i\eta(\omega) \mathcal{E}_+ e^{2i(\omega/c - k_s)z}, \quad (39b)$$

where

$$\alpha'(\omega) = \frac{a_0\gamma_e}{[|\Omega_d|^2 - \Delta'\Delta'_R]^2 + \gamma_e^2 \Delta_R'^2} \times \left\{ \Delta'_R (|\Omega_d|^2 - \Delta'\Delta'_R) + i[|\Omega_d|^2 \gamma_s + \gamma_e (\Delta_R'^2 + \frac{1}{8}S_{gs}^2)] \right\}, \quad (40a)$$

$$\eta(\omega) = \frac{a_0\gamma_e}{[|\Omega_d|^2 - \Delta'\Delta'_R]^2 + \gamma_e^2 \Delta_R'^2} \times \frac{1}{4} \left\{ |\Omega_d|^2 S_{gs} - \Delta'_R (2\Delta' S_{gs} + \Delta'_R S_g) + 2i\gamma_e \Delta'_R S_{gs} \right\}. \quad (40b)$$

The solution of Eqs. (39) is given by Eqs. (28), with

$$\delta\beta = \alpha'(\omega) + \frac{\omega}{c} - k_s, \quad s = \sqrt{\eta^2(\omega) - \delta\beta^2}.$$



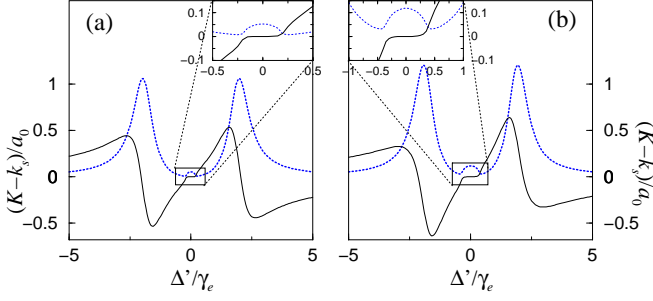


FIG. 6: Imaginary (blue dotted lines) and real (black solid lines) parts of  $K$  as a function of probe detuning  $\Delta'$  (resonant drive,  $\Delta' = \Delta'_R$ ), for  $\Omega_d = 2\gamma_e$ ,  $\gamma_s = 10^{-4}\gamma_e$  and  $\delta_s = 0$ . The ac Stark shift amplitudes are (a)  $\frac{1}{4}S_{gs} = 0.2\gamma_e$ , and (b)  $\frac{1}{4}S_{gs} = 0.4\gamma_e$ . Insets magnify the important frequency regions within the EIT window.

The corresponding dispersion relation  $K = k_s + is$ , with a resonant driving field  $\omega_d = \omega_{es} - \frac{1}{2}S_s$  ( $\Delta'_R = \Delta'$ ) and  $k_s \simeq \omega_{eg}/c$ , is shown in Fig. 6, where nonzero values of  $\text{Im} K$  within the EIT window signify the appearance of a PBG for the probe field.

In the vicinity of EIT resonance  $|\Delta'| < \Omega_d$  (assuming resonant drive,  $\Delta' = \Delta'_R$ ), Eqs. (40) reduce to  $\alpha'(\omega) \simeq \Delta'/v_g$  and  $\eta(\omega) \simeq S_{gs}/(4v_g)$ , and the dispersion relation can be approximated as

$$K - k_s \simeq i\sqrt{\left[\frac{S_{gs}}{4v_g}\right]^2 - \left[\frac{\Delta'}{v_g} - \frac{\delta_s}{c}\right]^2}. \quad (41)$$

For  $\delta_s = 0$ , a PBG for the probe field with frequencies in the range  $|\Delta'| < \frac{1}{4}|S_{gs}|$  is formed ( $\delta\omega_{\text{gap}} = \frac{1}{2}|S_{gs}|$ ). The peak value of  $\text{Im} K$  attained at the gap center  $\Delta'_{\text{gap}} = 0$  is  $\max(\text{Im} K) = |S_{gs}|/(4v_g)$ . Note that a PBG for the probe field exists only when  $S_{gs} \equiv S_g - S_s \neq 0$ , i.e., the ac Stark shifts  $S_g$  and  $S_s$  of the lower atomic levels  $|g\rangle$  and  $|s\rangle$  induced by an off-resonant standing wave field should be different. It is clear that in order to minimize absorption, the ac Stark modulation of the Raman resonance should be accommodated with the EIT window,  $\frac{1}{2}|S_{gs}| \lesssim \delta\omega_{\text{tw}}$  [7]. This, in turn, restricts the peak reflectivity of the medium to  $\max(\text{Im} K) \lesssim \sqrt{a_0/8L}$ .

Figure 7 shows the reflection, transmission and absorption spectra for the probe field for the two values of  $S_{gs}$  used in Fig. 6. In Fig. 7(a) with  $\frac{1}{4}S_{gs} = 0.2\gamma_e$ , the peak of the reflection coefficient  $R$  at  $\Delta' = 0$  is about 80% and the absorption  $A$  is 18%. Even though increasing  $S_{gs}$  results in broader band gap and larger values of  $\text{Im} K$ , as seen in Fig. 6(b) with  $\frac{1}{4}S_{gs} = 0.4\gamma_e$ , the corresponding reflection coefficient of Fig. 7(b) is smaller,  $R \simeq 67\%$ , due to the increased absorption  $A \simeq 33\%$ . Physically, this can be understood by recalling (see Fig. 2 and [9, 10]) that in the vicinity of EIT resonance, the dispersion (which determines  $\text{Im} K$ ) scales linearly with detuning  $\Delta_R$  (i.e., with  $S_{gs}$ ), while absorption scales quadratically with  $S_{gs}$ . Therefore, by choosing smaller values of the ac Stark modulation amplitudes  $S_{gs}$ , and taking simultaneously

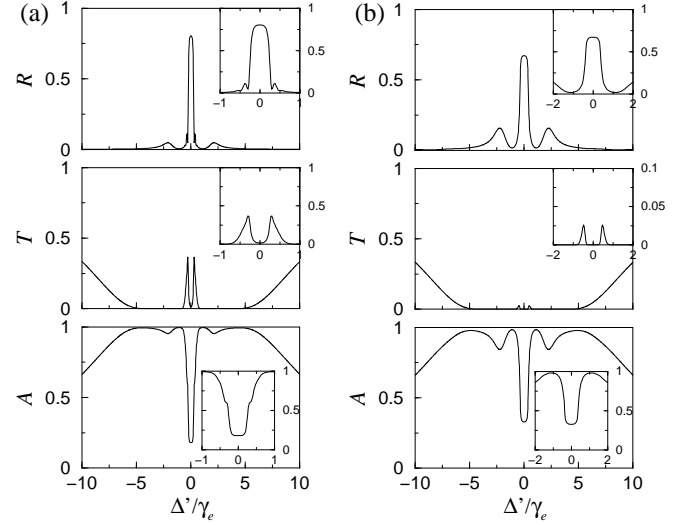


FIG. 7: Reflection  $R$ , transmission  $T$  and absorption  $A$  spectra for the probe field in a thermal EIT atomic medium subject to spatially periodic ac Stark modulation of the Raman resonance. The medium length is  $L \simeq 200 \mu\text{m}$  (optical depth  $2a_0L = 100$ ), and all the other parameters are same as in Fig. 6(a) and (b), respectively. Insets magnify the important frequency regions within the EIT window.

longer medium, to compensate for reduced  $\text{Im} K$ , will yield larger values of the reflection coefficient at the center of the PBG which will however shrink in width.

## V. CONCLUSIONS AND OUTLOOK

Equation (22) or (25) characterize the spectral properties of electromagnetically induced transparency (EIT) in an atomic medium, as illustrated in Fig. 2. Looking at that figure, one could come up with three possible ways for achieving spatially-periodic index modulation  $\delta n(\omega; z) \simeq \frac{1}{2}\text{Re} \chi(\omega; z)$  resulting in a photonic band gap (PBG) in the vicinity of EIT resonance  $|\Delta_R| < \Omega_d, \gamma_e$ , where the susceptibility is approximately given by

$$\chi(\omega; z) \simeq 2 \frac{c}{\omega} \frac{\sigma_0 \varrho \gamma_e}{|\Omega_d|^2} \Delta_R.$$

(i) To periodically modulate the medium density  $\varrho = \varrho(z)$  by trapping cold atoms in an optical lattice, as proposed and studied here. (ii) To periodically shift the EIT spectrum  $\Delta_R = \Delta_R(z)$  in a thermal atomic ensemble by using spatially periodic ac Stark shift of the Raman transition induced by off-resonant standing-wave field, as proposed in [7] and reviewed in Sec. IV above. (iii) To use a standing-wave drive field  $\Omega_d = \Omega_d(z)$  resulting in a spatially periodic modulation of the bandwidth of the EIT and the associated with it dispersion slope, as studied in [8].

The novel scheme proposed here possesses a number of advantageous properties. These include: (a) Greatly



reduced coherence relaxation on the two-photon Raman transition due to the elimination of atomic diffusion and collisions causing probe absorption. (b) Simultaneous enhancement of the refractive index modulation and the resulting Bragg reflection due to the tight localization of the atoms. (c) Tunability of the position and the width of the PBG within and beyond the EIT resonance. These properties can be employed for achieving very efficient nonlinear interactions between weak quantum fields and realization of deterministic quantum logic with single photons, as described in [14].

Finally, it would be interesting to explore the possibility of achieving a 2D PBGs for the probe field in the medium of cold atoms trapped in an optical lattice. The ultimate goal would be to explore structures with appropriately engineered defects which may allow for strong localization and waveguiding of light. Such defects can easily be implemented in the laboratory experiments by simply focusing a resonant laser onto the desired lattice sites, which will release (evaporate) the corresponding

atoms from the trap. In contrast, the microfabrication of desired defects in solid-state photonic crystal structures [2] involves complicated and time consuming growth and lithographic techniques which are often not easily reconfigurable and reproducible. Thus tunable photonic band gaps in optical lattices may serve to experimentally test various proposed structures in a simple, quickly reproducible way.

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